Prospect Theoretic Analysis of Anti-jamming Communications in Cognitive Radio Networks

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***Abstract*—An anti-jamming communication game between a cognitive radio enabled secondary user (SU) and a cognitive radio enabled jammer is considered, in which end-user decision making is modeled using prospect theory (PT). More specifically, the interactions between a user and a smart jammer (i.e., their respective choices of transmission probability) are formulated as a game under the assumption that end-user decision making under uncertainty does not follow the traditional objective assumptions stipulated by expected utility theory (EUT), but rather follows the subjective deviations specified by PT. Under the assumption that the capacity of the system is governed by the primary user activity, the Nash equilibria of the game are characterized under various conditions and the impact of the players' subjectivity (deviation from EUT behavior) on the SU's throughput is measured. Simulation results show that the subjective view of an SU tends to exaggerate the jamming probabilities and decreases its transmission probability, thus reducing the average throughput. On the other hand, the subjectivity of a jammer tends to reduce its jamming probability and thus increases the SU throughput.**

I. INTRODUCTION

Cognitive radio networks (CRNs) in the unlicensed bands are vulnerable to jamming attacks due to the open access and broadcast nature of transmission in these bands. Specifically, jammers can send signals in the hope of blocking the ongoing transmissions of secondary users (SUs) and thus performing denial of service attacks. With the recent advances in cognitive radio technology, smart jammers that have flexible control over their strategies such as jamming frequencies and signal strength can deteriorate CRNs. This has motivated extensive research studies on jamming [1]–[5]. Further, as both secondary users and jammers have autonomy and flexible control over their unlicensed band transmissions, game theory has emerged as a powerful tool to investigate anti-jamming radio communications [6]–[8].

Traditional game theory assumes that all the players are rational and uninfluenced by real-life perception. Consequently, most existing game theoretic studies on jamming are based on the assumption that both secondary users and jammers make decisions according to their expected utilities. However, this assumption deviates from real-life decision-making under uncertainty and traditional expected utility theory (EUT) cannot explain the deviations due to end-user subjectivity such as illustrated by Allais's paradox [9]. For example, we consider a two-option experiment with A: to win \$10 and B: to win \$0.01 and \$20 each with a probability 1/2. Due to the subjective nature of human decision-making, experimental results have shown that most people choose option A, although B leads to a higher expected utility [9], [10].

Therefore, prospect theory (PT) has been proposed to provide a user-centric view to address this issue, which applies a probability weighting effect to transform the objective probabilities into subjective probabilities [9], [10]. This Nobel prize winning theory was originally proposed for monetary transactions to explain the fact that people usually over-weigh low probability bad outcomes and under-weigh their favorable outcomes with high probabilities, which EUT fails to explain. This theory also explains the framing effect, i.e., players take into account the relative gains regarding a reference point rather than the final asset position, and the fact that losses loom larger than gains. While PT was originally developed to model and explain decision-making in monetary transactions, it has recently found widespread use in many contexts: social sciences [11]–[13], communication networks [14]–[19] and smart energy management [20], [21].

In this paper, we apply prospect theory to analyze anti-jamming communications in cognitive radio networks from a user-centric viewpoint. More specifically, we formulate the interactions between a smart jammer and a secondary user with mixed transmission strategies as a PT-based game, and apply Prelec's probability weight functions [22] to model the subjectivity of both players in the transmission. If controlled by a subjective owner, an SU emphasizes the potential jamming loss and chooses its transmission strategy to avoid it, while a jammer tends to overweigh its loss resulting from jamming on an unused channel.

We analyze the Nash equilibria (NE) of the PT-based anti-jamming communication game, in which the utility functions of both players are based on the SU's throughput. The SU's

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throughput results from the players' actions, as well as the network state defined as the channel capacity available to the SU resulting from both the primary user (PU) activity and the radio channel state. The Nash equilibria (NE) of the game are characterized under various conditions and the impact of the players' subjectivity (deviation from EUT behavior) on the SU's throughput is measured. Simulation results are presented to evaluate the impact of player subjectivity on the performance of the anti-jamming communication game.

The remainder of the paper is organized as follows. We briefly review related work in Section II and present the PT-based anti-jamming communication game in Section III. We analyze the NE strategies in the game under various scenarios in Section IV and provide simulation results in Section V. Finally, we conclude in Section VI.

II. Related Work

Game theoretic study of anti-jamming communications in wireless networks has attracted a lot of attention recently. In [7], a jamming game was analyzed for radio transmission in which the opponent's type is unknown and the corresponding Nash equilibrium was presented. Power control against a smart jammer was investigated following a Stackelberg game approach in [1]. A Bayesian jamming game was formulated for orthogonal frequency-division multiplexing systems in [23] and slotted Aloha systems in [24]. A channel state information feedback game against jammers was analyzed for a multi-channel system in [25].

Anti-jamming transmissions are an important consideration for cognitive radio networks in the unlicensed bands. While a SU's transmissions have to avoid an incumbent PU's transmissions, the SUs themselves are open to interference between them including jamming. Thus game theoretic studies of jamming in CRNs have been undertaken for various aspects of this problem. For example, spectrum sensing and channel access were analyzed based on a dynamic Bayesian game theory approach in [2]. The equilibrium of the jamming game in a CRN with incomplete information was evaluated in [3]. A game theoretic study of the transmission of data and control information was performed in [26], while a power control game in a multi-channel CRN against jamming was analyzed in [27]. The selection of the control channel against jammers was investigated using game theory in [4]. However, none of the above works have taken into account the subjectivity of end-users (SU and jammer) into consideration; and this is the focus of this paper.

III. PT-based Anti-jamming Communication Game

In this section, we formulate the anti-jamming communication game in a slotted-time single-channel cognitive radio network, while the analysis can be extended to the multi-channel case in the future. This game consists of two cognitive radio enabled players: a smart jammer and a secondary user whose goal is to send messages to its destination node without interfering with the primary user in the cognitive radio network. Each player can control its transmission autonomously and might hold a subjective view when making its decision in the static game.

$x = S, J$	Player x (SU or jammer)
a_x	Action of Player x
N_x	Number of actions of x minus 1
$\mathbf{p}_x = [p_{x,m}]_{0\le m\le N_x}$	Mixed strategy of Player x
$p_{x,m}$	Prob. for x to choose action m
$C_{x,m}$	Cost of action m to Player x
$u_x(m,n)$	Instant payoff of Player x
β_x	Utility ratio of Player x
U_x^{PT}	PT-based utility of Player x
U_x^{EUT}	EUT-based utility of x
α_x	Objective weight of Player x
$w_x(p)$	Subjective prob. weight func. of x
ς	State of the CRN
Θ_x	Payoff matrix of Player x
Ξ	Throughput of the SU

TABLE I

SUMMARY OF SYMBOLS AND NOTATION.

We assume that each player can access the channel with a given probability and denote the action of the SU by $a_S \in \{0, 1, \cdots, N_S\}$, where its transmission probability is given by a_S/N_S and $N_S(\geq 1)$ is the quantization level of the SU's channel access rate. For example, the SU is idle in the time slot if $a_S = 0$, while it transmits for sure if $a_S = N_S$. Similarly, we denote the action of the jammer by $a_J \in \{0, 1, \cdots, N_J\}$, i.e., the jammer blocks the channel with a probability a_J/N_J.

We first consider a noncooperative static game with pure strategies. More specifically, this game consists of two players: the SU with an action $a_S = m$ and a smart jammer with an action $a_J = n$. The network state denoted by ς is defined as the channel capacity available to the secondary user resulting from the the primary user's activity and the channel conditions. Therefore, the SU's throughput is given by $\frac{m}{N_S}\left(1-\frac{n}{N_J}\right)\varsigma$.

Let $u_x \triangleq [u_x(m,n)]_{0\leqslant m\leqslant N_S, 0\leqslant n\leqslant N_J}$ represent the instantaneous utility matrix of Player x, $x = S, J$, where $u_x(m,n)$ denotes the instantaneous utility of Player x with $a_S = m$ and $a_J = n$. We assume that it is a zero-sum game, in which the secondary user's utility $u_S(m,n)$ is based on its throughput, its transmission cost $C_{S,m}$ and the cost of the jammer $C_{J,n}$. Thus the instantaneous payoffs are given by

$$\begin{aligned} u_S(m,n) &= -u_J(m,n) \\ &= \frac{m}{N_S}\left(1-\frac{n}{N_J}\right)\varsigma - C_{S,m} + C_{J,n}, \end{aligned} \tag{1}$$

where $C_{x,a}$ is the cost of action a to Player x. Note that instead of being restricted to the zero-sum game given by (1), our work can be easily extended to the other cases with different utility functions.

Next, we extend the analysis to the mixed-strategy game, where each player chooses its action with randomness in order to fool its opponent. In the mixed-strategy game, both players take actions over their action sets according to their strategies given by $\mathbf{p}_J \triangleq [p_{J,m}]_{0\le m\le N_J}$ and $\mathbf{p}_S \triangleq [p_{S,m}]_{0\le m\le N_S}$, where $p_{x,m} \triangleq \Pr(a_x = m)$, $\forall x = J, S$.

Let $U_S^{EUT}(\mathbf{p}_S, \mathbf{p}_J)$ denote the expected utility of SU averaged over all the action realizations following $\mathbf{p}_S$ and $\mathbf{p}_J$. Assuming independent action strategies, we can write the expected utility of the SU and jammer, respectively, as

$$U_S^{EUT}(\mathbf{p}_S, \mathbf{p}_J) \triangleq \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} \Pr(a_S = m)\Pr(a_J = n) u_S(m,n)$$
$$= \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} p_{S,m} p_{J,n} u_S(m,n) \tag{2}$$
$$U_J^{EUT}(\mathbf{p}_S, \mathbf{p}_J) \triangleq \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} \Pr(a_S = m)\Pr(a_J = n) u_J(m,n)$$
$$= \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} p_{S,m} p_{J,n} u_J(m,n). \tag{3}$$

It is clear that for the zero-sum game with (1), we have $U_S^{EUT}(\mathbf{p}_S, \mathbf{p}_J) = -U_J^{EUT}(\mathbf{p}_S, \mathbf{p}_J)$.

We now apply prospect theory to formulate the anti-jamming communication and apply Prelec's probability weight function [22] to model the user subjectivity. In the PT-based game, each player holds a subjective view over the random action of its opponent and tends to over-weigh the unfavorable behavior of its opponent. More specifically, let $w_x(p)$ denote the probability weight function of player x, which is defined as the subjective probability for this player to weigh the outcome with a probability p. According to Prelec's work in [22], the weighting function can be chosen as

$$w_x(p) = \exp\left(-\left(-\ln p\right)^{\alpha_x}\right), \tag{4}$$

where $\alpha_x \in (0,1]$ is the objective weight and decreases with the player's subjective evaluation distortion. It is clear that this function is S-shaped and asymmetrical, with values ranging from 0 to 1. An objective player is a special case with $\alpha_x = 1$ and thus by (4) $w_x(p|\alpha_x = 1) = p$.

Instead of relying on the expected utility in (2) and (3), a subjective player x selects its action strategy according to the probability weight function $w_x(p)$. Therefore, a subjective SU or smart jammer chooses its channel access probability to maximize its prospect theory-based utility denoted by $U_x^{PT}(\mathbf{p}_S, \mathbf{p}_J)$, which is given by

$$U_S^{PT}(\mathbf{p}_S, \mathbf{p}_J) \triangleq \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} \Pr(a_S = m) w_S\left(\Pr(a_J = n)\right) u_S(m,n)$$
$$= \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} p_{S,m} w_S\left(p_{J,n}\right) u_S(m,n) \tag{5}$$
$$U_J^{PT}(\mathbf{p}_S, \mathbf{p}_J) \triangleq \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} w_J\left(\Pr(a_S = m)\right) \Pr(a_J = n) u_J(m,n)$$
$$= \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} w_J\left(p_{S,m}\right) p_{J,n} u_J(m,n). \tag{6}$$

It is clear that the PT-based utility of an objective SU or jammer (i.e., $\alpha_x = 1$) is the same as its EUT-based utility, i.e.,

$$U_x^{PT}(\mathbf{p}_S, \mathbf{p}_J|\alpha_x = 1) = U_x^{EUT}(\mathbf{p}_S, \mathbf{p}_J), \quad x = J, S. \tag{7}$$

For ease of reference, we summarize the commonly used notation in Table 1.

IV. Nash Equilibrium in the PT-based Anti-jamming Communication Game

We consider the Nash equilibrium in the PT-based anti-jamming communication game, which is denoted by ($\mathbf{p}_S^*$, $\mathbf{p}_J^*$), where $\mathbf{p}_x^* = [p_{x,m}^*]_{0 \leqslant m \leqslant N_x}$. In the NE of the mixed-strategy game, each player chooses its action strategy to maximize its own subjective utility function, $U_x^{PT}(\mathbf{p}_S, \mathbf{p}_J)$ by (5) and (6), with its opponent choosing the NE strategy. Therefore, each NE strategy is a best response to all other strategies in that equilibrium. We can obtain an NE of the game by the following:

$$\begin{array}{ll} \mathbf{p}_S^* & = \arg\max_{\mathbf{p}_S} U_S^{PT}(\mathbf{p}_S, \mathbf{p}_J^*) \\ \mathbf{p}_J^* & = \arg\max_{\mathbf{p}_J} U_J^{PT}(\mathbf{p}_S^*, \mathbf{p}_J) \\ s.t. & \sum_{m=0}^{N_J} p_{J,m} = 1 \\ & \sum_{m=0}^{N_S} p_{S,m} = 1 \\ & \mathbf{p}_S \succeq \mathbf{0}, \mathbf{p}_J \succeq \mathbf{0}. \end{array} \tag{8}$$

For convenience of denotation, we use Θ_S and Θ_J to represent the payoff matrixes for the SU and jammer, respectively, with

$$\Theta_x \triangleq [u_x(m,n)]_{0 \le m \le N_S, 0 \le n \le N_J}, \quad x = J, S, \tag{9}$$

where $u_x(m,n)$ given by (1) is a parameter known by both players in the game.

Theorem IV.1. *An NE of the PT-based anti-jamming communication game, if one exists, is given by*

$$\left\{ \begin{array}{l} \Theta_S \left[w_S\left(p_{J,k}^*\right)\right]_{0 \leqslant k \leqslant N_J}^T = \lambda_S \left[1, \cdots, 1\right]^T \\ \Theta_J \left[w_J\left(p_{S,k}^*\right)\right]_{0 \leqslant k \leqslant N_S}^T = \lambda_J \left[1, \cdots, 1\right]^T \\ \sum_{m=0}^{N_S} p_{S,m}^* = 1 \\ \sum_{m=0}^{N_J} p_{J,m}^* = 1 \\ \lambda_S > 0 \\ \lambda_J > 0 \end{array} \right. , \tag{10}$$

where λ_S and λ_J are Lagrange parameters whose values are obtained from solving for the constraints (i.e., the sum of the transmission probabilities is 1) at the NE solution in (10).

Proof: We define F_S and by (5) simplify it into

$$F_S \triangleq U_S^{PT}(\mathbf{p}_S, \mathbf{p}_J^*) - \lambda_S \sum_{m=0}^{N_S} p_{S,m}$$
$$= \sum_{m=0}^{N_S}\sum_{n=0}^{N_J} p_{S,m} w_S\left(p_{J,n}^*\right) u_S(m,n) - \lambda_S \sum_{m=0}^{N_S} p_{S,m}. \tag{11}$$

According to the Karush-Kuhn-Tucker (KKT) optimality conditions on (8), by (11), we have

$$\frac{\partial F_S}{\partial p_{S,k}} = \sum_{n=0}^{N_J} u_S(k,n) w_S\left(p^*_{J,n}\right) - \lambda_S = 0, \tag{12}$$

with $0 \le k \le N_S$, which can be rewritten as

$$\begin{bmatrix} u_S(0,0) & u_S(0,1) & \cdots & u_S(0,N_J) \\ u_S(1,0) & u_S(1,1) & \cdots & u_S(1,N_J) \\ & & \cdots & \\ u_S(N_S,0) & u_S(N_S,1) & \cdots & u_S(N_S,N_J) \end{bmatrix}$$
$$\cdot \begin{bmatrix} w_S\left(p^*_{J,0}\right) \\ w_S\left(p^*_{J,1}\right) \\ \cdots \\ w_S\left(p^*_{J,N_J}\right) \end{bmatrix} = \Theta_S \begin{bmatrix} w_S\left(p^*_{J,0}\right) \\ w_S\left(p^*_{J,1}\right) \\ \cdots \\ w_S\left(p^*_{J,N_J}\right) \end{bmatrix} = \lambda_S \begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}. \tag{13}$$

Similarly, by applying the KKT optimality condition and using (6), we have $p^*_{S,k}$ for $0 \leqslant k \leqslant N_S$, and thus obtain (10). ■

Corollary IV.2. *If the payoff matrixes Θ_S and Θ_J are positive definite and have a rank of $N+1$, with $N = N_J = N_S$, an NE of the PT-based anti-jamming communication game is given by*

$$\begin{cases} p^*_{S,k} = \exp\left(-\left(-\ln\left(\lambda_J \sum_{m=0}^N \left[\Theta_J^{-1}\right]_{m,k}\right)\right)^{\frac{1}{\alpha_J}}\right), \\ p^*_{J,k} = \exp\left(-\left(-\ln\left(\lambda_S \sum_{m=0}^N \left[\Theta_S^{-1}\right]_{m,k}\right)\right)^{\frac{1}{\alpha_S}}\right), \\ \forall k = 0, \cdots, N \\ \sum_{m=0}^N p^*_{S,m} = 1, \quad \sum_{m=0}^N p^*_{J,m} = 1, \end{cases} \tag{14}$$

where $[A]_{i,j}$ is the (i,j)-th element of the matrix A.

Proof: We consider the case in which the payoff matrixes Θ_S and Θ_J are positive definite and have a rank of $N+1$, with $N = N_J = N_S$. Since $\lambda_S > 0$ and $w_S(\cdot) \ge 0$, if Θ_S is positive definite and has a rank $N+1$, Θ_S has an inverse matrix that is also positive definite.

In this case, the linear equations (10) can be solved and we can simplify its first line into

$$w_S(p^*_{J,k}) = \lambda_S \sum_{m=0}^N [\Theta_S^{-1}]_{m,k}, k = 0, \cdots, N. \tag{15}$$

By integrating (4) into (15), we have

$$p^*_{J,k} = \exp\left(-\left(-\ln\left(\lambda_S \sum_{m=0}^N \left[\Theta_S^{-1}\right]_{m,k}\right)\right)^{\frac{1}{\alpha_S}}\right). \tag{16}$$

Similarly, we have $p^*_{S,k}$ for $0 \leqslant k \leqslant N$ and thus obtain (14). ■

For the case with two transmission quantization levels, i.e., $N = 1$, the anti-jamming communication game can be simplified. Define

$$\beta_x = \frac{u_x(1,0) - u_x(0,0)}{u_x(0,1) - u_x(1,1)}, \qquad x = J, S. \tag{17}$$

In this case, the NE, $\mathbf{p}^*_S = [p^*_{S,0}, 1 - p^*_{S,0}]$ and $\mathbf{p}^*_J = [p^*_{J,0}, 1 - p^*_{J,0}]$ can be obtained by the following:

Corollary IV.3. *If $\min(\beta_S, \beta_J) > 1$, the PT-based anti-jamming communication game with $N = 1$ has a unique NE, which is given by*

$$\left(\ln\left(\frac{1}{1-p^*_{S,0}}\right)\right)^{\alpha_J} - \left(\ln\left(\frac{1}{p^*_{S,0}}\right)\right)^{\alpha_J} + \ln\beta_J = 0 \tag{18}$$

$$\left(\ln\left(\frac{1}{1-p^*_{J,0}}\right)\right)^{\alpha_S} - \left(\ln\left(\frac{1}{p^*_{J,0}}\right)\right)^{\alpha_S} + \ln\beta_S = 0 \tag{19}$$

Proof: As $N = 1$, by (13), we have

$$\begin{aligned} &(u_S(0,0) - u_S(1,0))\, w_S(p^*_{J,0}) \\ &= (u_S(1,1) - u_S(0,1))\, w_S(p^*_{J,1}). \end{aligned} \tag{20}$$

According to the definition of the subjective probability weight function in (4), we take the logarithm of (20) and introduce the notation $\beta_S = (u_S(1,0) - u_S(0,0)) / (u_S(0,1) - u_S(1,1))$, yielding

$$\left(-\ln\left(p^*_{J,1}\right)\right)^{\alpha_S} - \left(-\ln\left(p^*_{J,0}\right)\right)^{\alpha_S} + \ln\beta_S = 0. \tag{21}$$

By replacing $p^*_{J,1}$ with $1 - p^*_{J,0}$, we obtain

$$\left(\ln\left(\frac{1}{1-p^*_{J,0}}\right)\right)^{\alpha_S} - \left(\ln\left(\frac{1}{p^*_{J,0}}\right)\right)^{\alpha_S} + \ln\beta_S = 0.$$

Similarly, we can obtain $p^*_{S,0}$ as in (18).

Next, we prove the uniqueness of $p^*_{S,0}$ when $\beta_J > 1$. Note that this condition is true when the marginal utility of transmitting is greater than that of not transmitting. For simplicity, we define $f(x) \triangleq \left(\ln\left(\frac{1}{x}\right)\right)^{\alpha_J}$, which is monotonically decreasing with x. If $\beta_J > 1$, by (18) we have $f(p^*_{S,0}) > f(p^*_{S,1})$, thus yielding $0 < p^*_{S,0} < p^*_{S,1} < 1$. As $p^*_{S,0} + p^*_{S,1} = 1$, we have $0 < p^*_{S,0} < 0.5$. It is easy to verify that the derivative of $g(x) \triangleq f(1-x) - f(x)$ is positive if $0 < x < 0.5$, i.e., $dg(x)/dx = f'(1-x) - f'(x) > 0$, indicating a monotonically increasing function. Therefore, (18) has a unique solution. Similarly, when $\beta_S > 1$, we can also prove the uniqueness of $p^*_{J,0}$. ■

Corollary IV.4. *The NE of the anti-jamming communication game consisting of two objective players with $N = 1$ is given by*

$$p^*_{S,0} = \frac{u_J(1,1) - u_J(0,1)}{u_J(1,1) - u_J(0,1) + u_J(0,0) - u_J(1,0)} \tag{22}$$

$$p^*_{J,0} = \frac{u_S(1,1) - u_S(0,1)}{u_S(1,1) - u_S(0,1) + u_S(0,0) - u_S(1,0)}. \tag{23}$$

Proof: As both players are objective, i.e., $\alpha_S = \alpha_J = 1$, by (18), we have

$$\ln\left(\frac{1}{1-p^*_{S,0}}\right) - \ln\left(\frac{1}{p^*_{S,0}}\right) + \ln\beta_J = 0, \tag{24}$$

which can be further simplified by (17) as

$$p^*_{S,0} = \frac{1}{1+\beta_J} = \frac{u_J(1,1) - u_J(0,1)}{u_J(1,1) - u_J(0,1) + u_J(0,0) - u_J(1,0)}. \quad (25)$$

Similarly, we can obtain $p^*_{J,0}$ as in (23). ∎

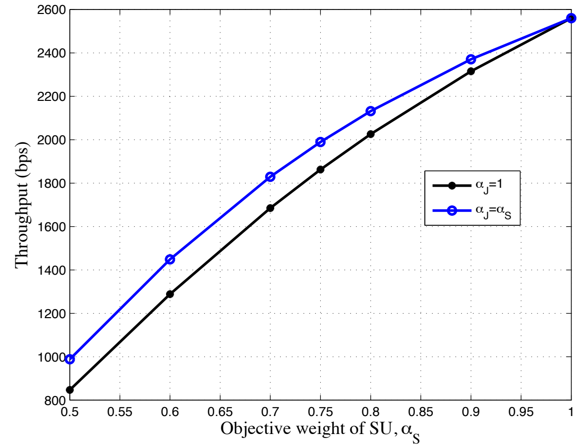
V. Simulation Results

We performed simulations to evaluate the impact of the users' objective weights α_x in (4) on the performance of the NE strategy described above in the PT-based anti-jamming communication game. We present the average utilities of both players, U_S^{EUT} and U_J^{EUT}, as given by (2) and (3). In addition, we also computed the average throughput of the SU, denoted with Ξ, which can be written as

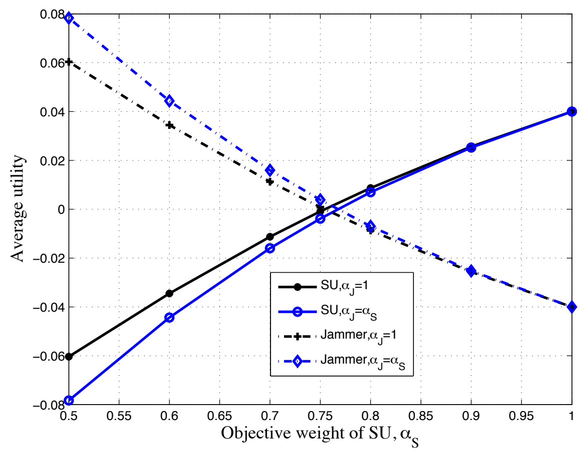
$$\Xi(\mathbf{p}_S, \mathbf{p}_J) = \sum_{m=0}^{N_S} \sum_{n=0}^{N_J} p_{S,m} p_{J,n} \frac{m}{N_S} \left(1 - \frac{n}{N_J}\right) \varsigma. \quad (26)$$

In the simulation, we assume that the network state $\varsigma = 16$ kbps, $C_{J,0} = C_{S,0} = 0$, $C_{J,1} = 0.05$ and $C_{S,1} = 0.2$. Therefore, by (1), the payoff matrices in this zero-sum game are given by $u_S = -u_J = [0, 0.05; 0.8, -0.15]$. The performance at the NE is presented in Fig. 1, showing that both the SU's throughput and utility increase with its objectivity (i.e., α_S). For instance, the SU's throughput increases from about 800 bps to 2580 bps, as α_S changes from 0.5 to 1, with $\alpha_J = 1$. The reason for this gain in throughput is that a subjective SU is less likely to transmit to avoid its highly unfavorable loss due to a jammed transmission. Consequently, the SU receives a higher average utility as shown in Fig. 1(b).

In addition, it is shown in Fig. 1 that the SU's transmission benefits from the jammer's subjective view. For a fixed α_S, a smaller value of α_J results in a higher throughput for the SU. This is because a subjective jammer is less likely to block the SU's transmission as it overweights its loss from wasteful jamming. On the other hand, this subjective view of the smart jammer saves its jamming power and thus increases its utility, which indicates a reduction in the SU's utility in this zero-sum game.



(a) Average throughput of SU.



(b) Average SU and jammer utilities.

Fig. 1. Performance of a SU and smart jammer vs. their objective weights (i.e., α_S and α_J) in an anti-jamming communication game in a cognitive radio network.

VI. Conclusion

In a cognitive radio network, while an SU's transmissions have to avoid an incumbent PU's transmissions, the SUs themselves are open to interference between them including jamming. We have formulated an anti-jamming communication game under the situation when both the SU and the smart jammer hold subjective views on their decision making. Using prospect theory to model end-user behavior, we derive transmission strategies for both the SU and the jammer. By analyzing the NE of the anti-jamming communication game, we have investigated the impact of the players' subjective view on the anti-jamming communication, which is ignored by the traditional expected utility theory. Simulation results have shown that by exaggerating the potential damages resulting from jamming, a subjective secondary user decreases its transmission rate and obtains a lower throughput. On the other hand, a subjective jammer is less likely to attack the SU and thus increases the latter's throughput. In the future, we plan to formulate anti-jamming communications over a long time period as a dynamic game, in which a smart jammer holds a subjective view on the random change of the SU's state.

References

[1] D. Yang, G. Xue, J. Zhang, A. Richa, and X. Fang, "Coping with a smart jammer in wireless networks: A Stackelberg game approach," *IEEE Trans. Wireless Commun.*, vol. 12, pp. 4038 – 4047, Aug. 2013.

[2] Q. Zhu, Z. Han H. Li, and T. Basar, "A stochastic game model for jamming in multi-channel cognitive radio systems," in *Proc. IEEE Int'l Conf. Commun.*, 2010, pp. 1 – 6.

[3] A. Gupta, A. Nayyar, C. Langbort, and T. Basar, "A dynamic transmitter-jammer game with asymmetric information," in *Proc. IEEE Annual Conf. Decision and Control*, 2012, pp. 6477 – 6482.

[4] B. F. Lo and I. F. Akyildiz, "Multiagent jamming-resilient control channel game for cognitive radio ad hoc networks," in *Proc. IEEE Int'l Conf. Commun.*, 2012, pp. 1821 – 1826.

[5] L. Xiao, H. Dai, and P. Ning, "Jamming-resistant collaborative broadcast using uncoordinated frequency hopping," *IEEE Trans. Inform. Forensics and Security*, vol. 7, pp. 297 – 309, Feb. 2011.

[6] L. Xiao, Y. Chen, W.S. Lin, and K.J.R. Liu, "Indirect reciprocity security game for large-scale mobile wireless networks," *IEEE Trans. Inform. Forensics and Security*, vol. 7, pp. 1368 – 1380, Aug. 2012.

[7] Y. Sagduyu, R. Berry, and A. Ephremides, "Jamming games in wireless networks with incomplete information," *IEEE Commun. Mag.*, vol. 49, pp. 112 – 118, Aug. 2011.

[8] K. Khalil and E. Ekici, "Multiple access game with a cognitive jammer," in *Proc. Asilomar Conf. Signals, Systems and Computers*, 2012, pp. 1383 – 1387.

[9] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk," *Econometrica*, vol. 47, pp. 263–291, 1979.

[10] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, vol. 5, pp. 297–323, 1992.

[11] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *J. Risk and uncertainty*, vol. 5, pp. 297–323, April 1992.

[12] S. Gao, E. Frejinger, and M. Ben-Akiva, "Adaptive route choices in risky traffic networks: A prospect theory approach," *Transportation Research Part C: Emerging Technologies*, vol. 18, pp. 727–740, May 2010.

[13] G. W. Harrison and E. E. Rutstrom, "Expected utility theory and prospect theory: One wedding and a decent funeral," *Experimental Economics*, vol. 12, pp. 133–158, Feb. 2009.

[14] T. Li and N. B. Mandayam, "Prospects in a wireless random access game," in *Proc. 46th Annual Conf. Information Sciences and Systems*, Mar. 2012.

[15] T. Li and N. B. Mandayam, "When users interfere with protocols: Prospect theory in wireless networks using random access and data pricing as an example," *IEEE Trans. Wireless Commun.*, pp. 1888 – 1907, April 2014.

[16] J. Yu, M. H. Cheung, and J. Huang, "Spectrum investment with uncertainty based on prospect theory," in *Proc. IEEE Int'l Conf. Commun.*, 2014.

[17] D. D. Clark, J. Wroclawski, K. R. Sollins, and R. Braden, "Tussle in cyberspace: defining tomorrows internet," *IEEE/ACM Trans. Networking*, vol. 13, pp. 462–475, Mar. 2005.

[18] Y. Wang, A. V. Vasilakos, Q. Jin, and J. Ma, "On studying relationship between altruism and the psychological phenomenon of self-deception in rational and autonomous networks," in *Proc. 32nd Int'l Conf. Distributed Computing Systems Workshops*, 2012.

[19] T. Okuda, T. Ichikawa, T. Ideguchi, and X. Tian, "A design method of local community network service systems with ad-hoc network technology," in *Proc. IEEE 70th Vehicular Technology Conf. Fall (VTC 2009-Fall)*, 2009.

[20] L. Xiao, N.B. Mandayam, and H.V. Poor, "Prospect theoretic analysis of energy exchange among microgrids," *IEEE Trans. Smart Grid*, under review.

[21] Y. Wang, W. Saad, N. B. Mandayam, and H. V. Poor, "Integrating energy storage into the smart grid: A prospect theoretic approach," in *Proc. IEEE Int'l Conf. Acoustics, Speech and Signal Processing*, 2014.

[22] D. Prelec, "The probability weighting function," *Econometrica*, pp. 497–528, 1998.

[23] A. Garnaev, Y. Hayel, and E. Altman, "A bayesian jamming game in an OFDM wireless network," in *Proc. Int'l Symp. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, 2012, pp. 41 – 48.

[24] L. Chen and J. Leneutre, "Fight jamming with jamming-a game theoretic analysis of jamming attack in wireless networks and defense strategy," *Computer Networks*, vol. 55, pp. 2259 – 2270, Jun. 2011.

[25] S. Wei, R. Kannan, V. Chakravarthy, and M. Rangaswamy, "CSI usage over parallel fading channels under jamming attacks: A game theory study," *IEEE Trans. Commun.*, vol. 60, pp. 1167 – 1175, April 2012.

[26] B. Wang, Y. Wu, K. J. R. Liu, and T. C. Clancy, "An anti-jamming stochastic game for cognitive radio networks," *IEEE J. Selected Areas Commun.*, vol. 29, pp. 877 – 889, Apr. 2011.

[27] Y. Wu, B. Wang, K. J. R. Liu, and T. Clancy, "Anti-jamming games in multi-channel cognitive radio networks," *IEEE J. Selected Areas Commun.*, vol. 30, pp. 4 – 15, Jan. 2012.